Calculation of CCT and Duv and Practical Conversion Formulae

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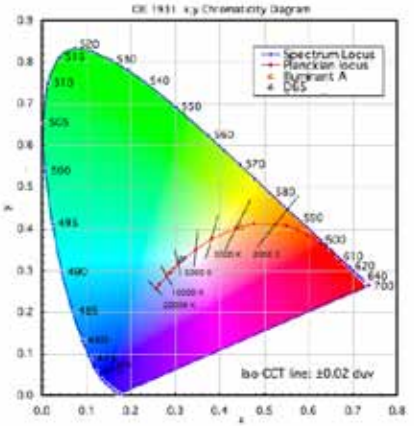
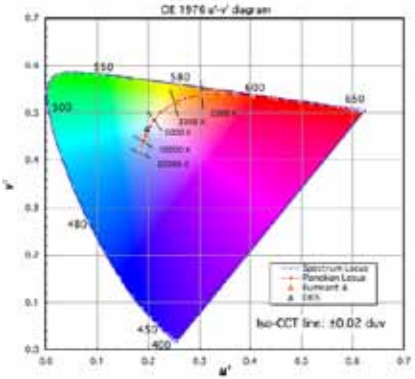
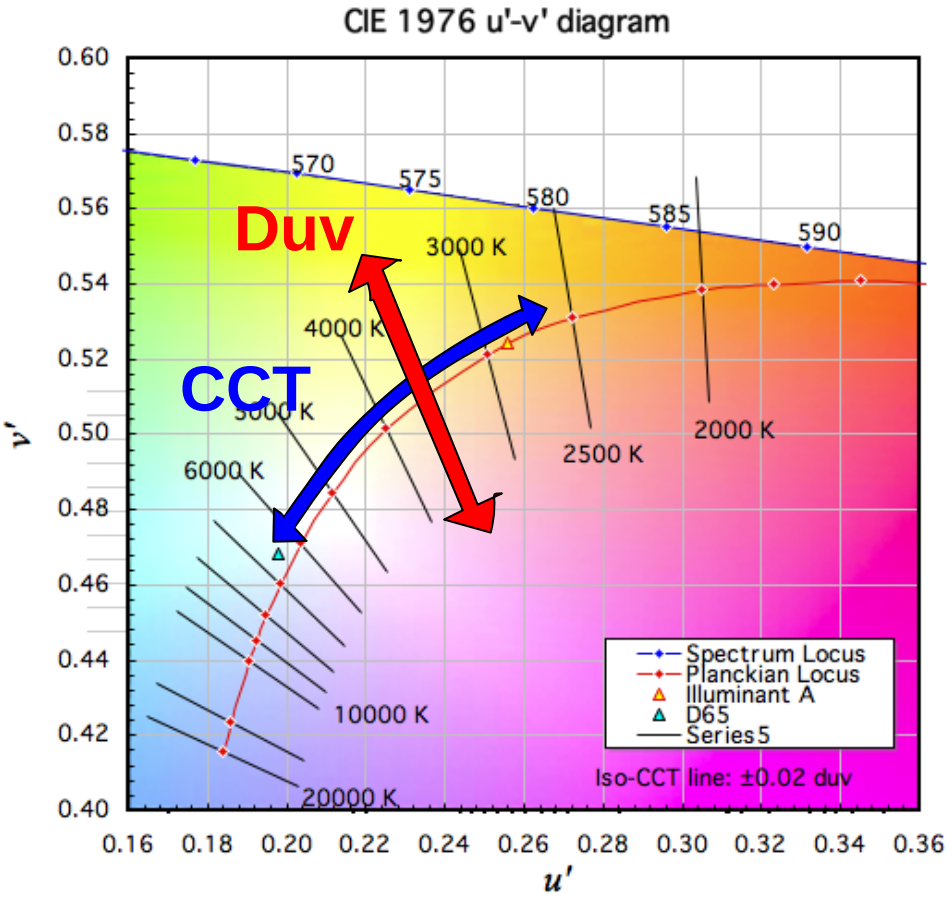
White Light Chromaticity

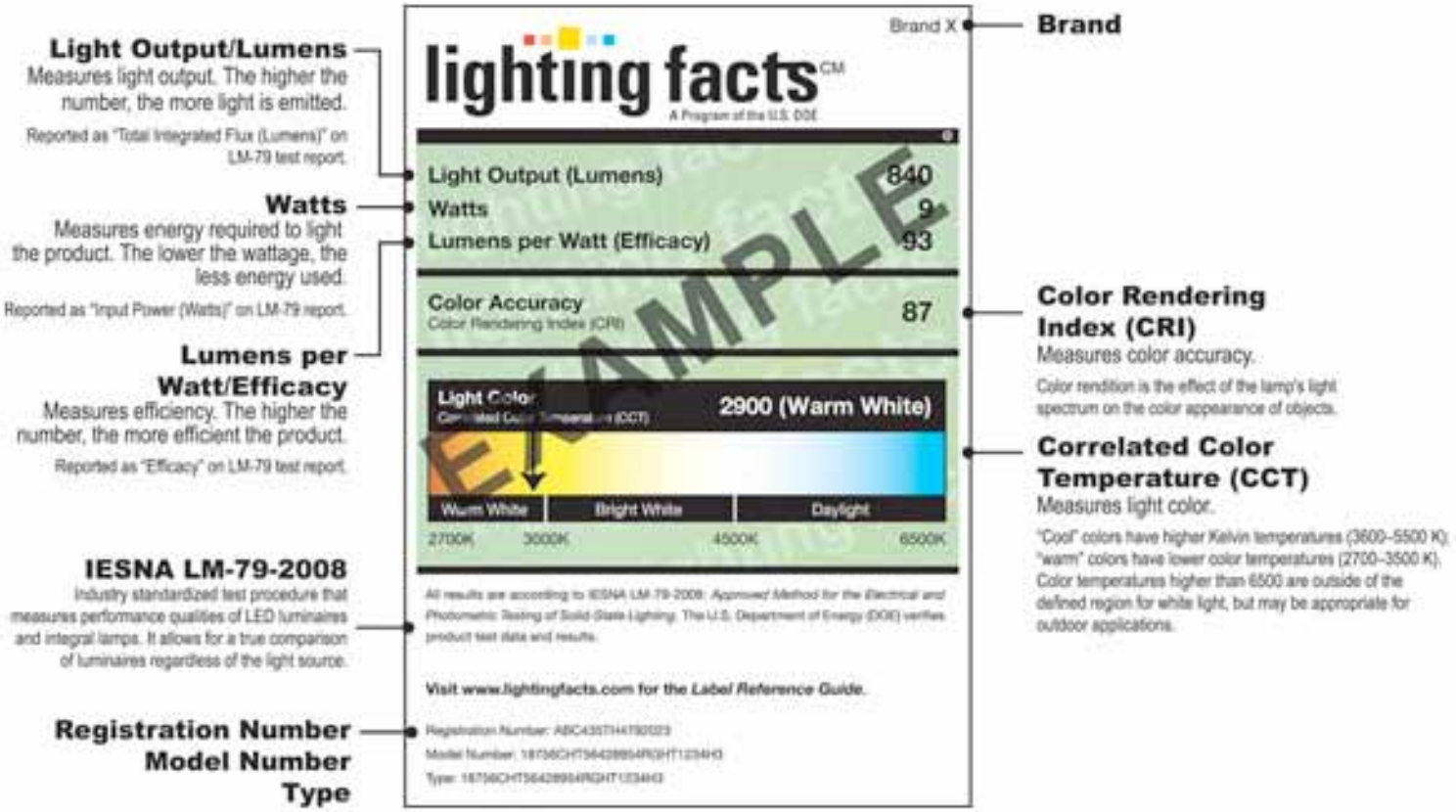






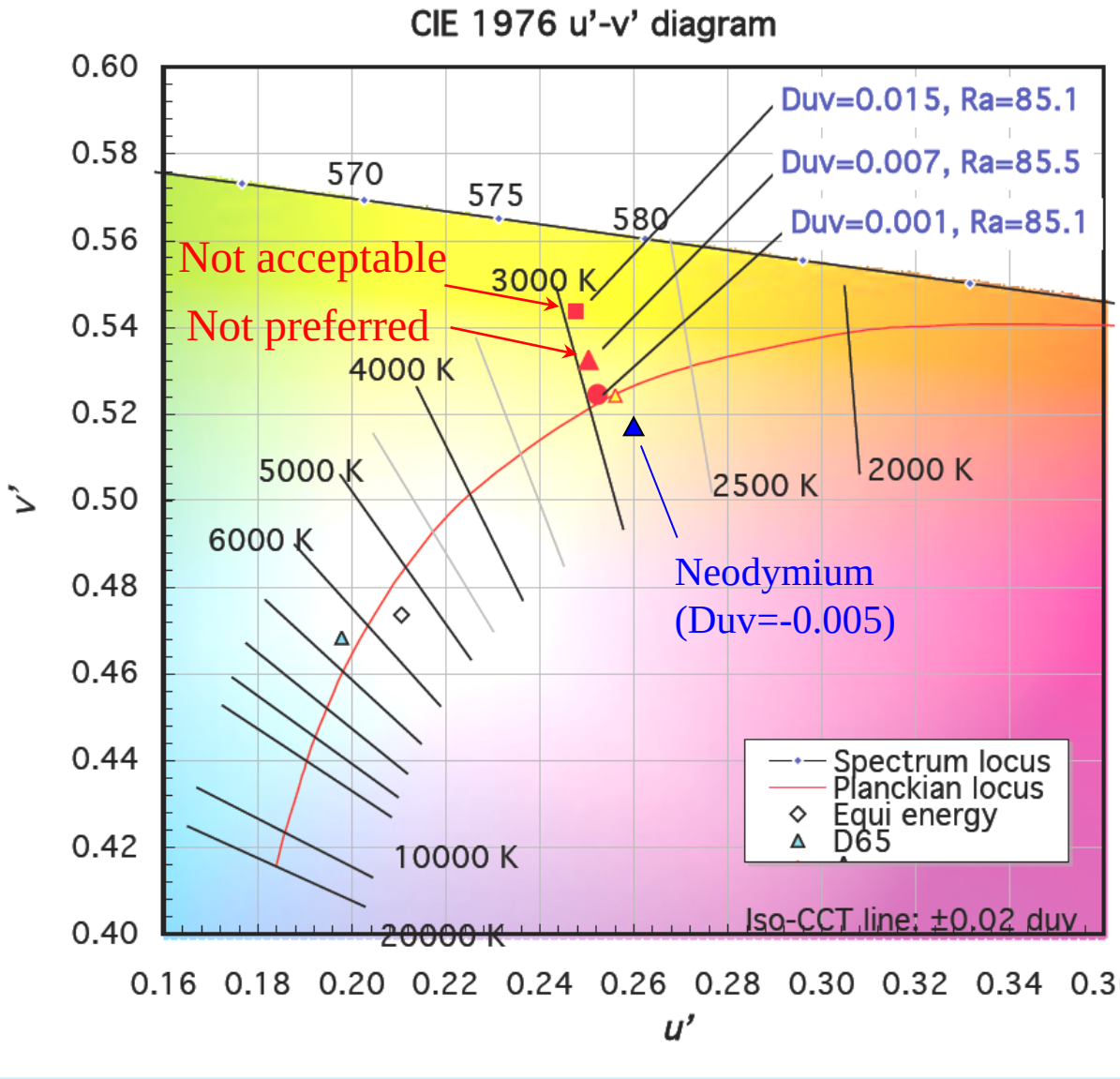
Duv often missing

Lighting Facts Label

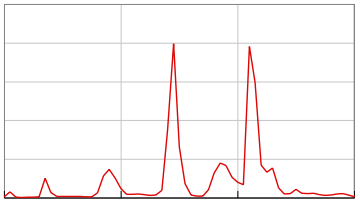


CCT and CRI do not tell the whole story of color quality

CCT and CRI do not tell the whole story



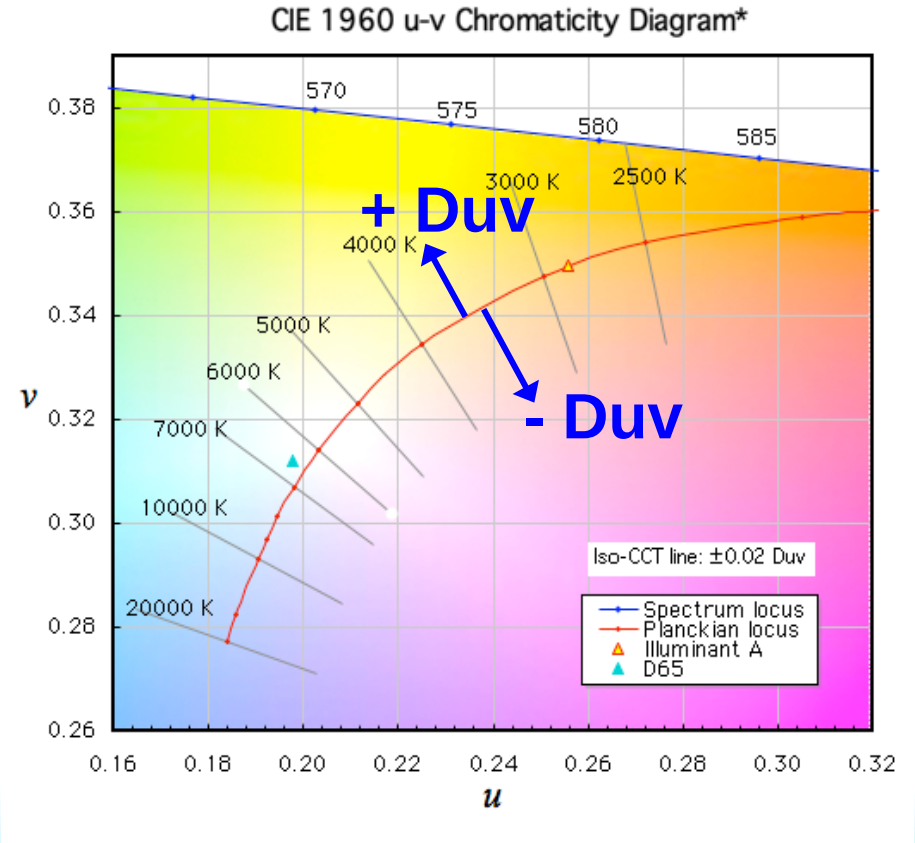
Triphosphor
FL simulation



Duv is another important dimension of chromaticity.

Duv defined in ANSI standard

Closest distance from the Planckian locus on the (*u'*, 2/3 *v'*) diagram, with + sign for above and - sign for below the Planckian locus. (ANSI C78.377-2008)



Symbol: D_{uv}

CCT and Duv can specify the chromaticity of light sources just like (*x*, *y*).

The two numbers (CCT, Duv) provides color information intuitively. (*x*, *y*) does not.

Duv needs to be defined by CIE.

ANSI C78.377-2008 Specifications for the chromaticity of SSL products

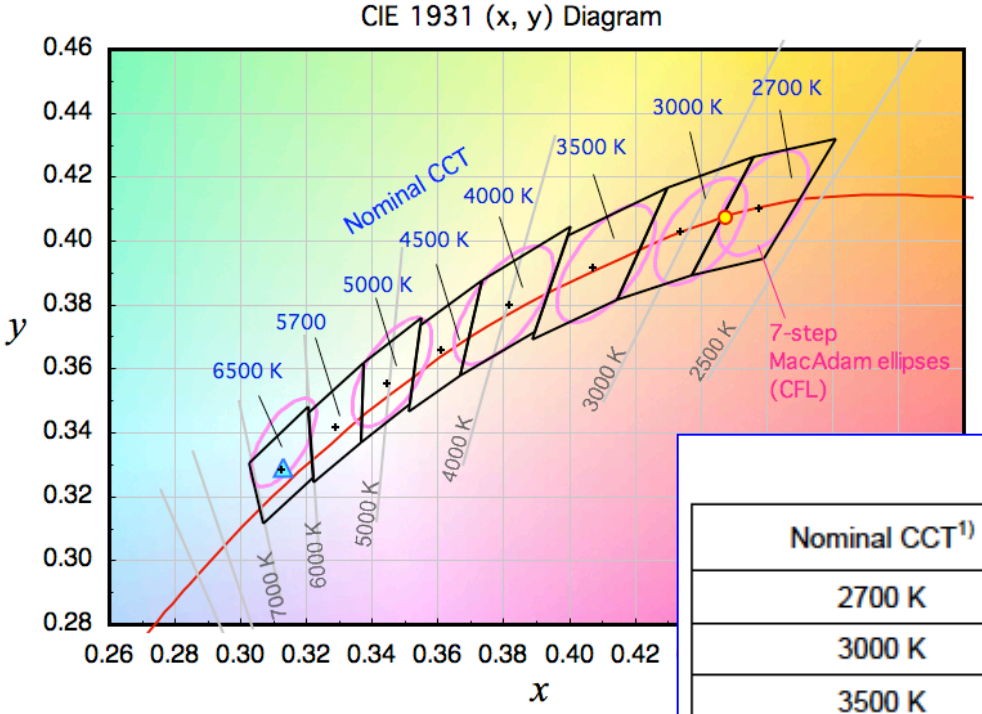
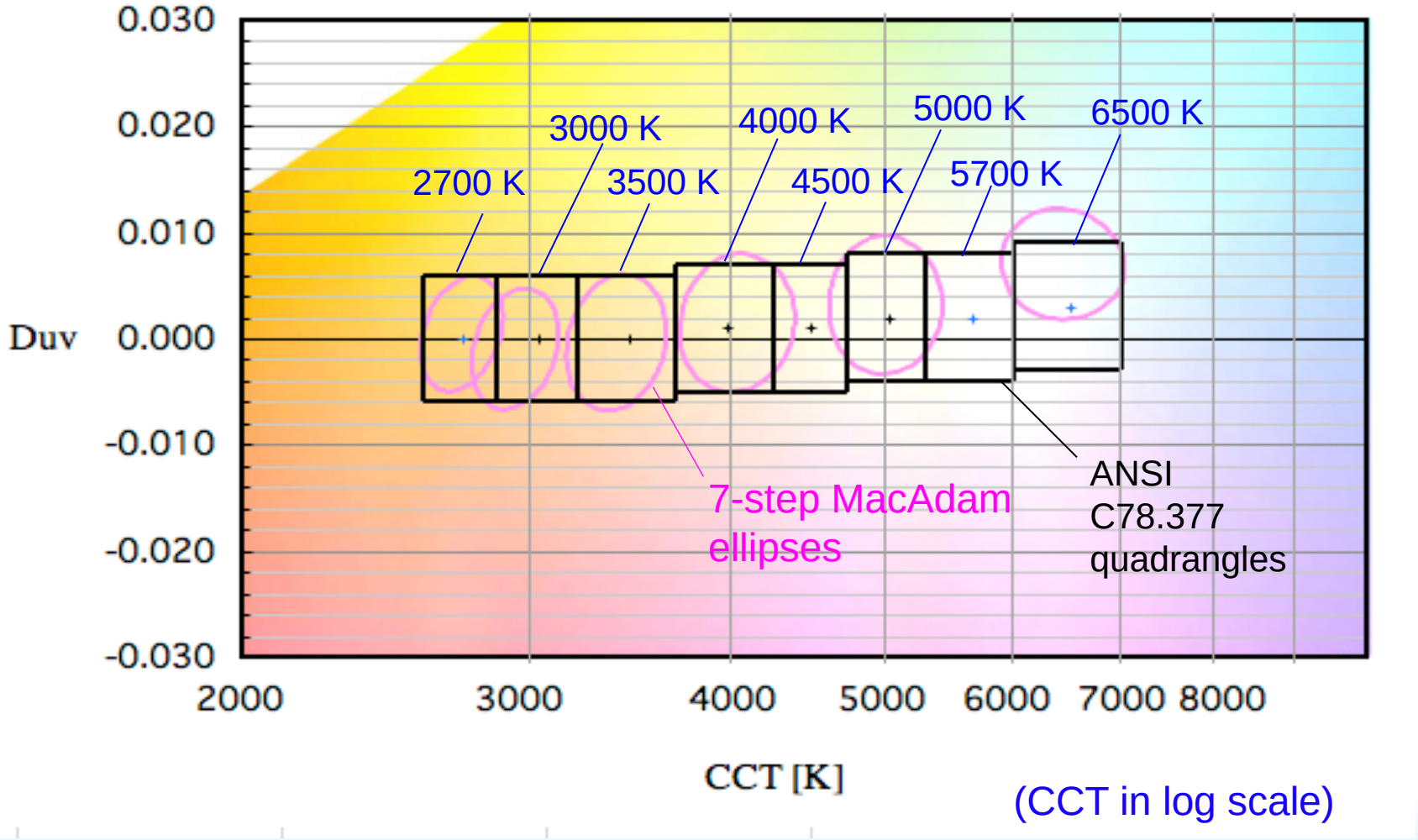


Table 1 - Nominal CCT Categories

Nominal CCT[1)]	Target CCT and tolerance (K)	Target Duv and tolerance
2700 K	2725 ± 145	0.000 ± 0.006
3000 K	3045 ± 175	0.000 ± 0.006
3500 K	3465 ± 245	0.000 ± 0.006
4000 K	3985 ± 275	0.001 ± 0.006
4500 K	4503 ± 243	0.001 ± 0.006
5000 K	5028 ± 283	0.002 ± 0.006
5700 K	5665 ± 355	0.002 ± 0.006
6500 K	6530 ± 510	0.003 ± 0.006
Flexible CCT (2700 - 6500 K)	T [2)] ± ΔT [3)]	D_{uv} [4)] ± 0.006

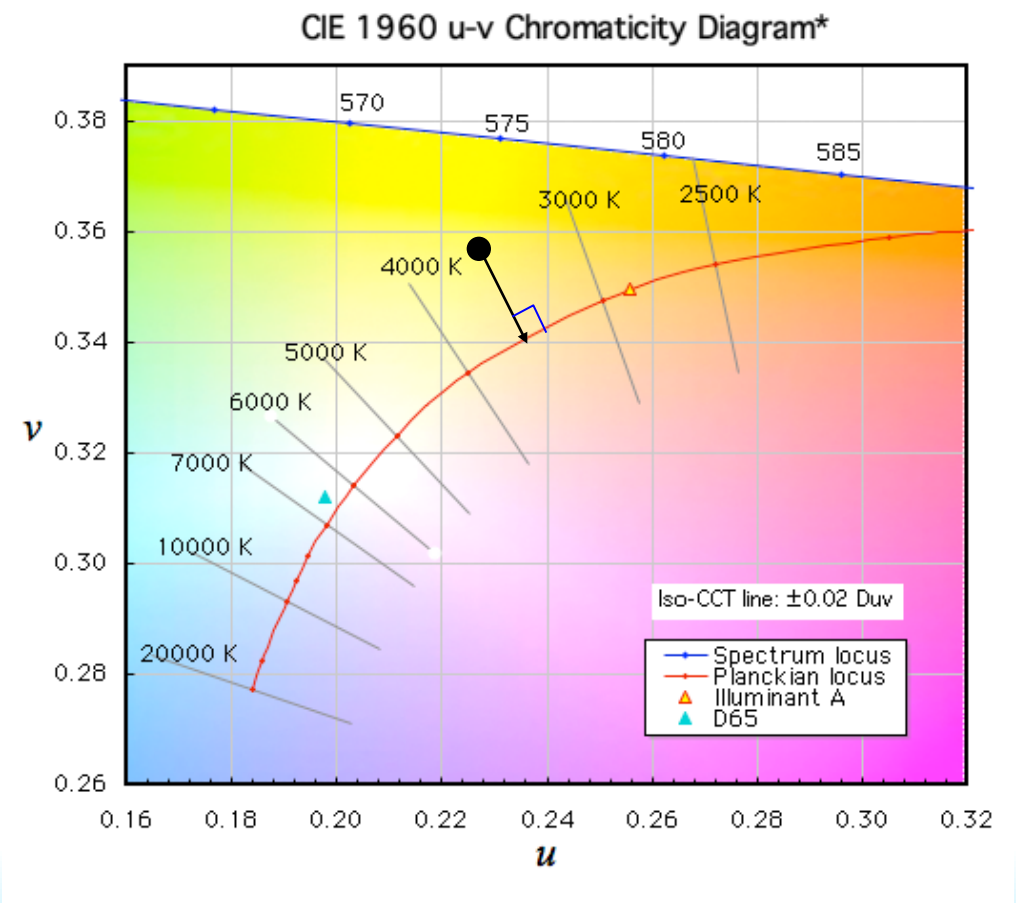
CCT- Duv chart



Correlated Color Temperature (CCT)

Temperature [K] of a Planckian radiator whose chromaticity is closest to that of a given stimulus on the CIE (*u′,2/3 v′*) coordinate.

(CIE 15:2004)



CCT is based on the CIE 1960 (*u, v*) diagram, which is now obsolete.

CCT is valid within distance 0.05 from the Planckian locus on the (*u'*, 2/3·*v'*) diagram. (CIE 15: 2004)

CIE 15:2004 Colorimetry, 3rd Edition

APPENDIX E. INFORMATION ON THE USE OF PLANCK'S EQUATION FOR STANDARD AIR

According to the Planck's law, the spectral radiance of a blackbody at thermodynamic temperature T [K] in a medium having index of refraction n is given by

$$L_{e,\lambda}(\lambda,T) = \frac{c_1 n^{-2} \lambda^{-5}}{\pi}\left[\exp\left(\frac{c_2}{n\lambda T}\right) - 1\right]^{-1} \qquad \text{(E.1)}$$

where $c_1 = 2\pi hc^2$, $c_2 = hc/k$, h is Planck's constant, c is the speed of light in vacuum, k is the ……

T should follow the current International Temperature Scale (ITS-90), therefore,

$c_2 = 1{,}4388 \times 10^{-2}$ m K.

……

Therefore, in the current recommendation in CIE 15:2004, colour temperature and correlated colour temperature are calculated using Equ. E.1 with n = 1 (exactly 1), thus no change from the previous practice. This recommendation may be subject to change in the future.

Robertson (1968)

Computation of Correlated Color Temperature and Distribution Temperature, Journal of the Optical Society of America, 58-11, 1968

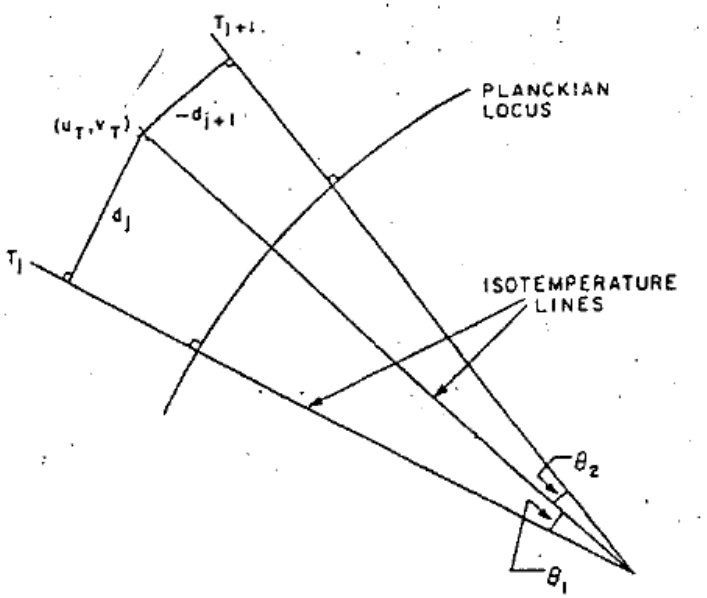


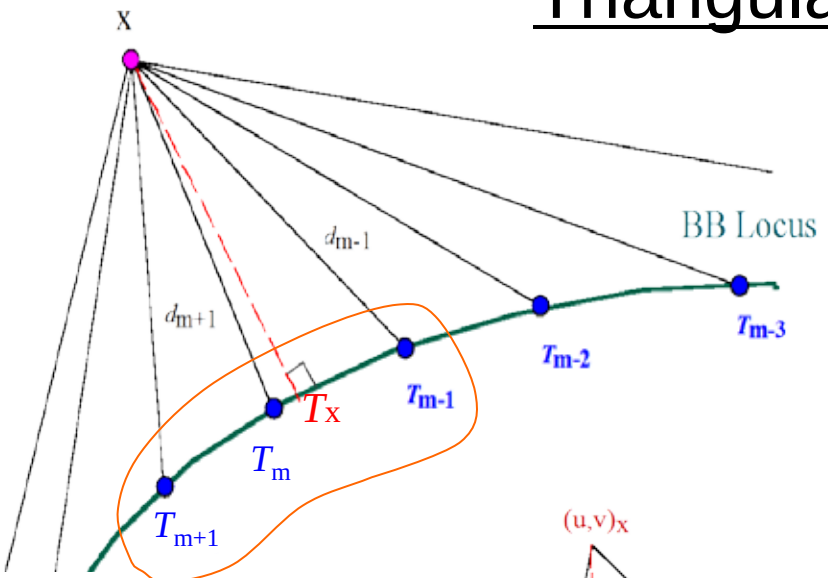
FIG. 1. Method of interpolation to find correlated color temperature.

TABLE III. Maximum errors of computed values of correlated color temperature, based on use of the 31 isotemperature lines listed in Table II.

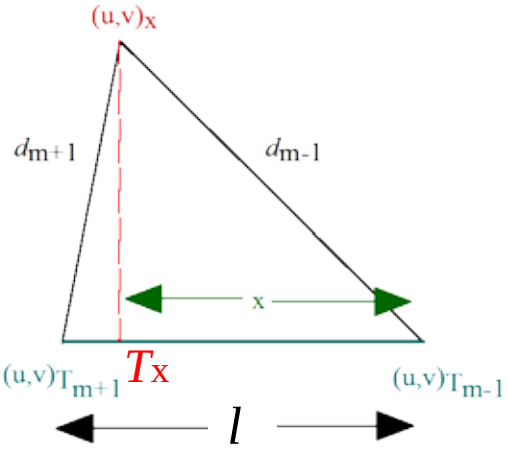
Range		Maximum error	
μrd	K	μrd	K
1–10	1 000 000–100 000	0.11	...
10–20	100 000–50 000	0.09	450
20–30	50 000–33 333	0.08	140
30–40	33 333–25 000	0.08	65
40–50	25 000–20 000	0.07	36
50–60	20 000–16 667	0.05	18
60–70	16 667–14 286	0.04	9.6
70–80	14 286–12 500	0.03	5.4
80–90	12 500–11 111	0.03	3.6
90–100	11 111–10 000	0.03	2.8
100–125	10 000–8 000	0.07	5.9
125–150	8 000–6 667	0.03	1.5
150–175	6 667–5 714	0.05	1.8
175–200	5 714–5 000	0.03	1.0
200–225	5 000–4 444	0.04	1.0
225–250	4 444–4 000	0.05	0.8
250–275	4 000–3 636	0.05	0.7
275–300	3 636–3 333	0.04	0.5
300–325	3 333–3 077	0.03	0.3
325–350	3 077–2 857	0.03	0.2
350–375	2 857–2 667	0.02	0.2
375–400	2 667–2 500	0.03	0.2
400–425	2 500–2 353	0.04	0.2
425–450	2 353–2 222	0.04	0.2
450–475	2 222–2 105	0.05	0.2
475–500	2 105–2 000	0.04	0.2
500–525	2 000–1 905	0.04	0.2
525–550	1 905–1 818	0.05	0.2
550–575	1 818–1 739	0.05	0.2
575–600	1 739–1 667	0.06	0.2

Direct approach (1) to calculate CCT and Duv

Triangular solution



CCT	*u*	*v*	distance d
2705	0.26228	0.35150	0.00772
2732	0.26109	0.35115	0.00650
2759	0.25992	0.35080	0.00529
2787	0.25876	0.35044	0.00411
2815	0.25761	0.35007	0.00297
2843	0.25647	0.34970	0.00193
2871	0.25535	0.34932	0.00122
2900	0.25425	0.34893	0.00144
2929	0.25315	0.34855	0.00232
2958	0.25207	0.34815	0.00337
2988	0.25100	0.34775	0.00446
3018	0.24995	0.34735	0.00556
3048	0.24890	0.34694	0.00666
3078	0.24788	0.34653	0.00776
3109	0.24686	0.34611	0.00885
3140	0.24585	0.34569	0.00994
3172	0.24486	0.34526	0.01101



(1) Create a table of CCT vs distance d_i to BB locus on (u,v) coodinate.

(2) Find the closest point in the table.

(3) Solve the triangle for the neighboring 2 points

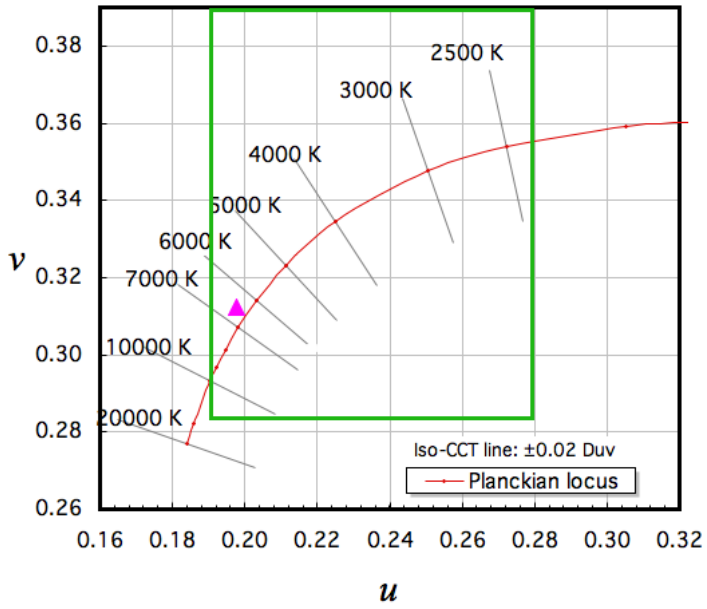
$$x = \frac{d_{m-1}^2 - d_{m+1}^2 + l^2}{2l}$$

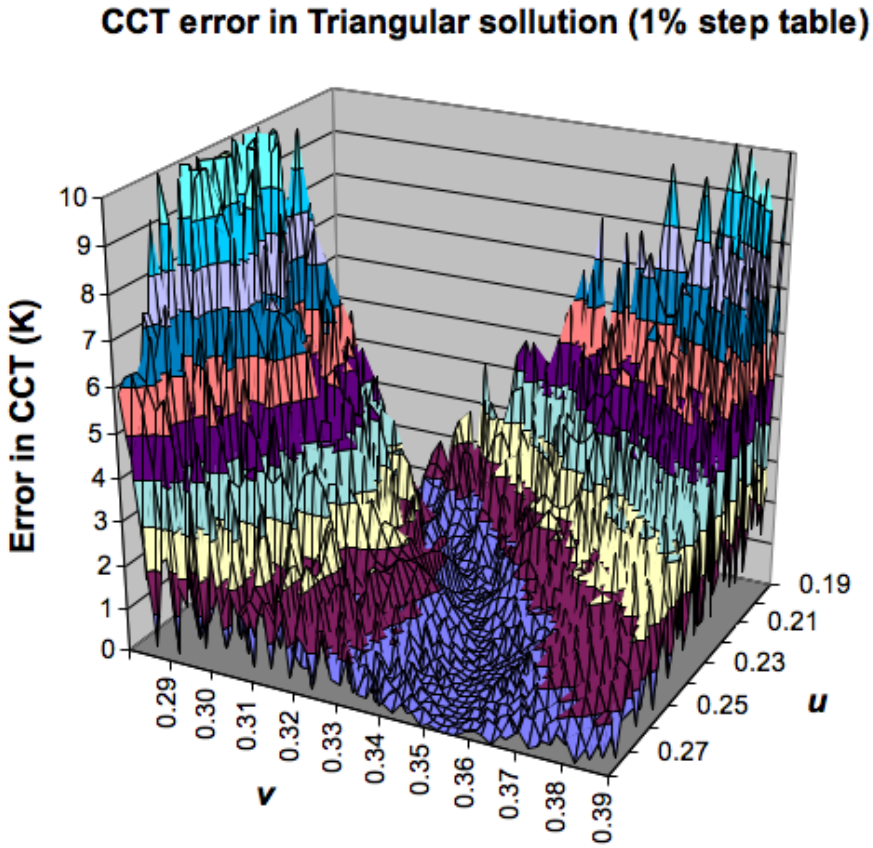
$$T_x = T_{m-1} + \left(T_{m+1} - T_{m-1}\right) \bullet \frac{x}{l}$$

$$D_{uv} = [\pm sign]\left(d_{m-1}^2 - x^2\right)^{1/2}$$

Use Planck's equation and color matching functions at 1 nm interval.

CCT Error in Triangular Solution

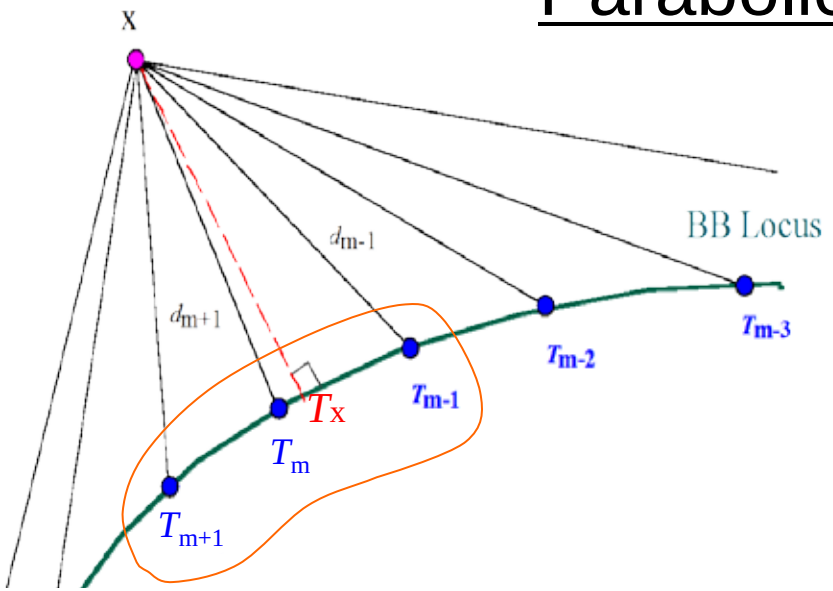




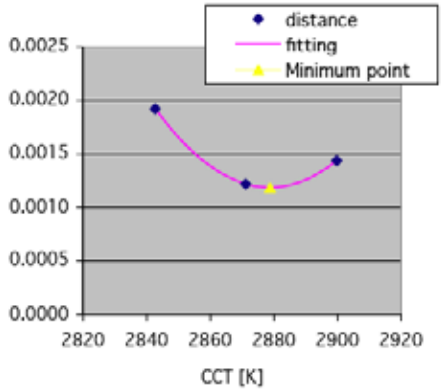
Error increases when the point is far from Planckian locus.

Direct approach (2) to calculate CCT and Duv

Parabolic solution



2705	0.26228	0.35150	0.00772
2732	0.26109	0.35115	0.00650
2759	0.25992	0.35080	0.00529
2787	0.25876	0.35044	0.00411
2815	0.25761	0.35007	0.00297
2843	0.25647	0.34970	0.00193
2871	0.25535	0.34932	0.00122
2900	0.25425	0.34893	0.00144
2929	0.25315	0.34855	0.00232
2958	0.25207	0.34815	0.00337
2988	0.25100	0.34775	0.00446
3018	0.24995	0.34735	0.00556
3048	0.24890	0.34694	0.00666
3078	0.24788	0.34653	0.00776
3109	0.24686	0.34611	0.00885
3140	0.24585	0.34569	0.00994
3172	0.24486	0.34526	0.01101



(1) Create a table of CCT vs distance d_i to BB locus on (u,v) coodinate.

(2) Find the closest point in the table.

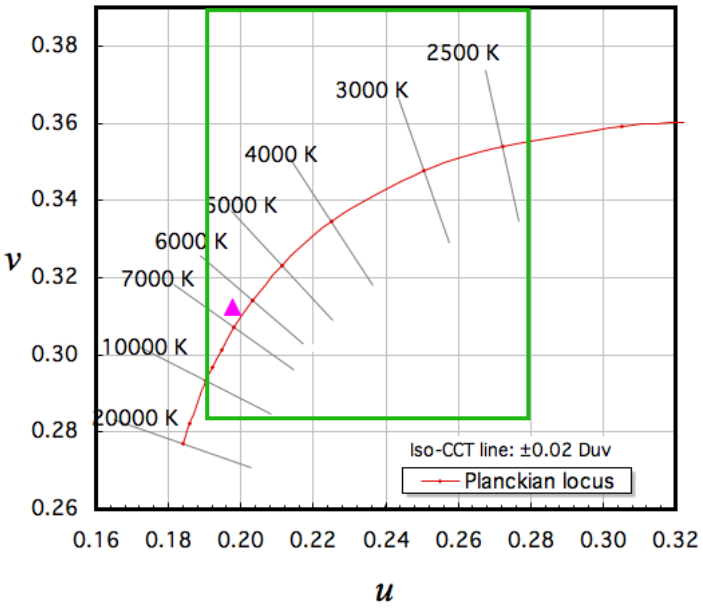
(3) Parabolic fit for the neighboring 3 points.

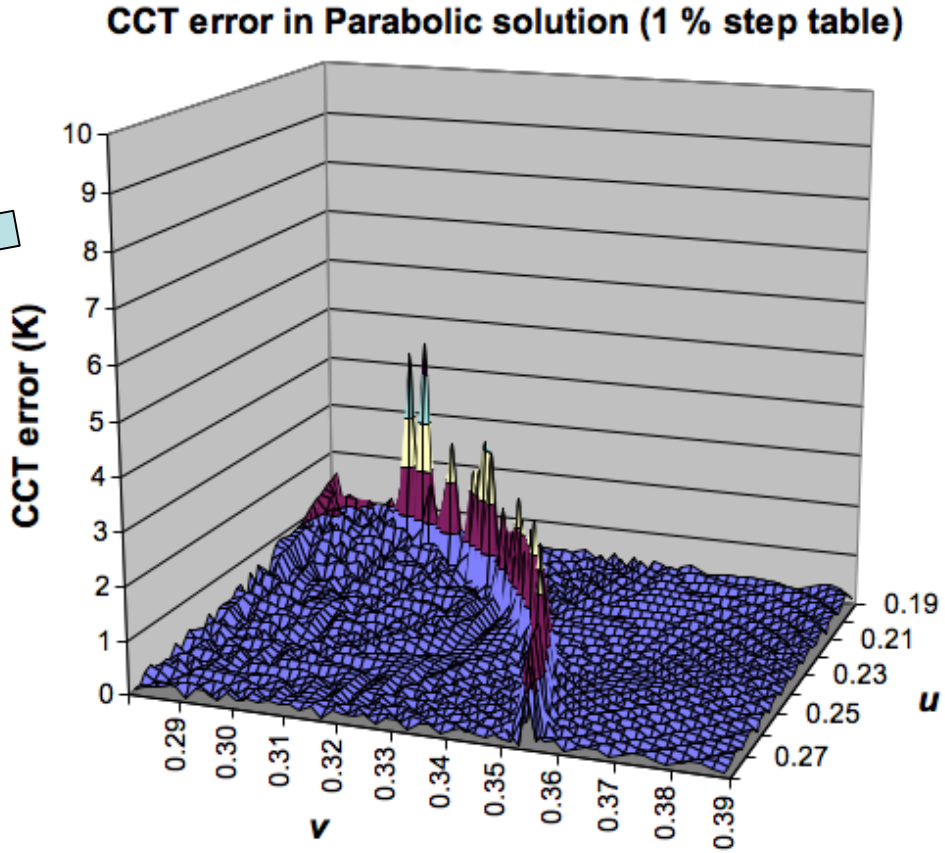
$$d(T) = aT^2 + bT + C$$

$$d(T)' = 2aT_x + b = 0 \quad \because T_x = \frac{-b}{2a}$$

$$D_{uv} = [\pm sign]\left(aT_x^{\,2} + bT_x + C\right)$$

CCT Error in Parabolic Solution



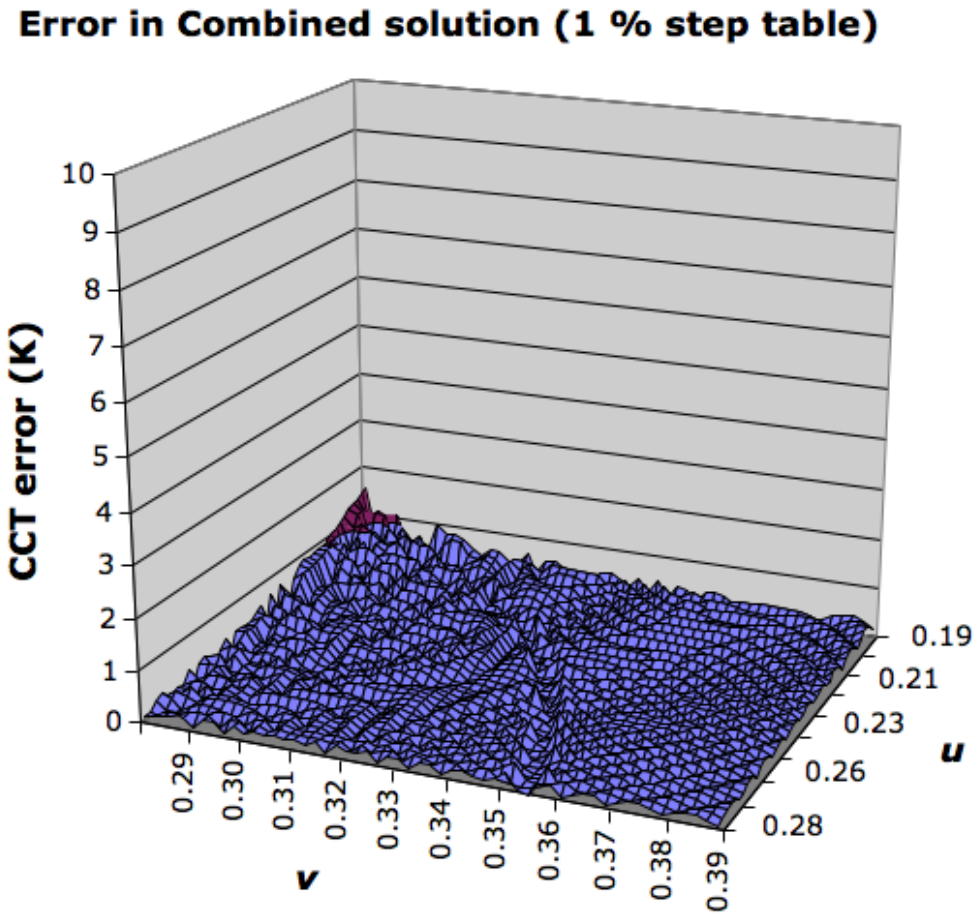


Much better, but the problem is on or very close to Planckian locus.

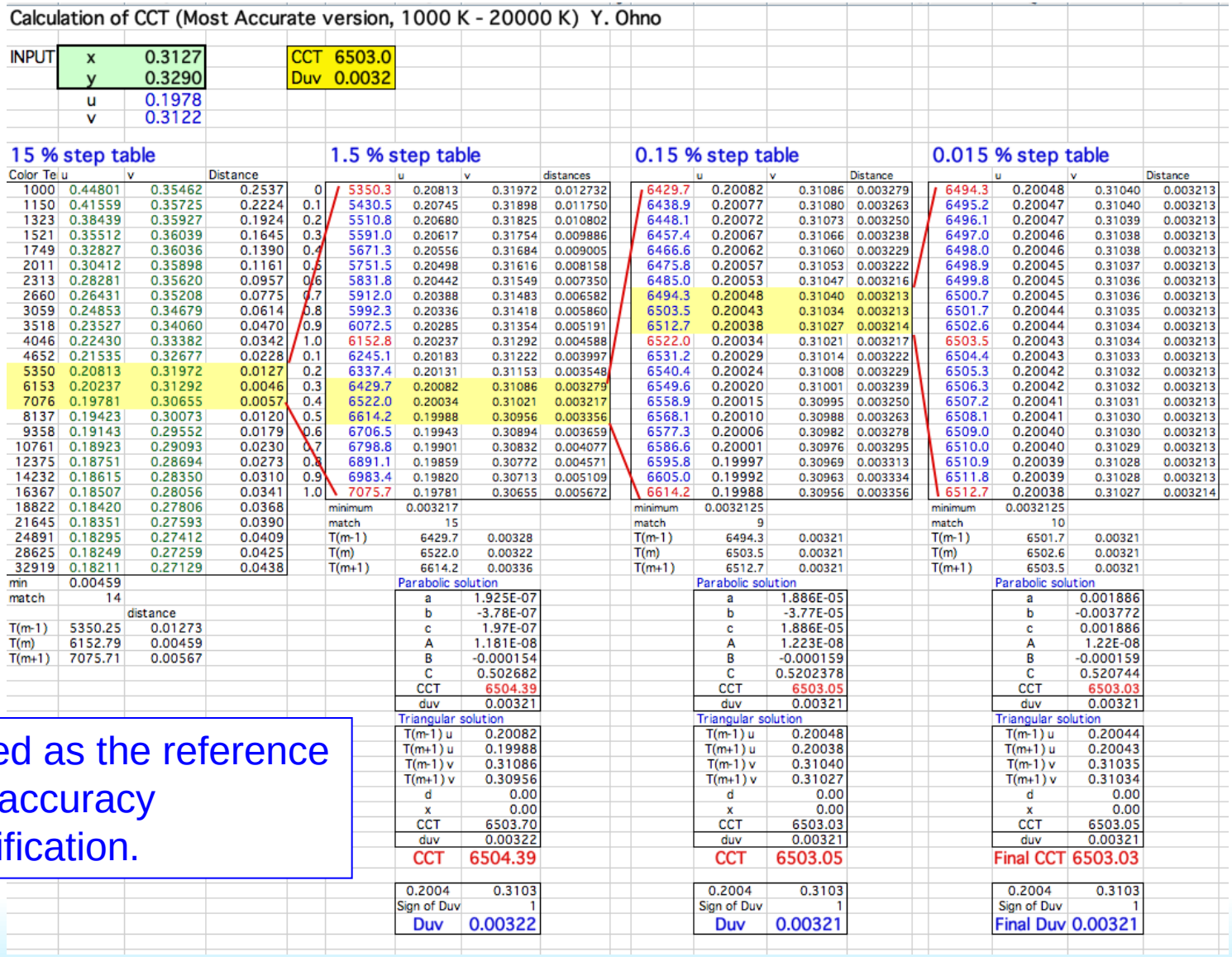
Combined Solution

Use Parabolic solution but, take the CCT of Triangular solution for

$$\left|D_{uv}\right| < 0.002$$



Most Accurate Version (cascade expansion)



Calculation of CCT (Most Accurate version, 1000 K - 20000 K) Y. Ohno

INPUT		
	x	0.3127
	y	0.3290
	u	0.1978
	v	0.3122

CCT	6503.0
Duv	0.0032

15 % step table

Color Te	u	v	Distance
1000	0.44801	0.35462	0.2537
1150	0.41559	0.35725	0.2224
1323	0.38439	0.35927	0.1924
1521	0.35512	0.36039	0.1645
1749	0.32827	0.36036	0.1390
2011	0.30412	0.35898	0.1161
2313	0.28281	0.35620	0.0957
2660	0.26431	0.35208	0.0775
3059	0.24853	0.34679	0.0614
3518	0.23527	0.34060	0.0470
4046	0.22430	0.33382	0.0342
4652	0.21535	0.32677	0.0228
5350	0.20813	0.31972	0.0127
6153	0.20237	0.31292	0.0046
7076	0.19781	0.30655	0.0057
8137	0.19423	0.30073	0.0120
9358	0.19143	0.29552	0.0179
10761	0.18923	0.29093	0.0230
12375	0.18751	0.28694	0.0273
14232	0.18615	0.28350	0.0310
16367	0.18507	0.28056	0.0341
18822	0.18420	0.27806	0.0368
21645	0.18351	0.27593	0.0390
24891	0.18295	0.27412	0.0409
28625	0.18249	0.27259	0.0425
32919	0.18211	0.27129	0.0438

min	0.00459	
match	14	
		distance
T(m-1)	5350.25	0.01273
T(m)	6152.79	0.00459
T(m+1)	7075.71	0.00567

1.5 % step table

	u	v	distances
0	5350.3	0.20813	0.31972	0.012732
0.1	5430.5	0.20745	0.31898	0.011750
0.2	5510.8	0.20680	0.31825	0.010802
0.3	5591.0	0.20617	0.31754	0.009886
0.4	5671.3	0.20556	0.31684	0.009005
0.5	5751.5	0.20498	0.31616	0.008158
0.6	5831.8	0.20442	0.31549	0.007350
0.7	5912.0	0.20388	0.31483	0.006582
0.8	5992.3	0.20336	0.31418	0.005860
0.9	6072.5	0.20285	0.31354	0.005191
1.0	6152.8	0.20237	0.31292	0.004588
0.1	6245.1	0.20183	0.31222	0.003997
0.2	6337.4	0.20131	0.31153	0.003548
0.3	6429.7	0.20082	0.31086	0.003279
0.4	6522.0	0.20034	0.31021	0.003217
0.5	6614.2	0.19988	0.30956	0.003356
0.6	6706.5	0.19943	0.30894	0.003659
0.7	6798.8	0.19901	0.30832	0.004077
0.8	6891.1	0.19859	0.30772	0.004571
0.9	6983.4	0.19820	0.30713	0.005109
1.0	7075.7	0.19781	0.30655	0.005672

minimum	0.003217	
match	15	
T(m-1)	6429.7	0.00328
T(m)	6522.0	0.00322
T(m+1)	6614.2	0.00336

Parabolic solution

a	1.925E-07
b	-3.78E-07
c	1.97E-07
A	1.181E-08
B	-0.000154
C	0.502682
CCT	6504.39
duv	0.00321

Triangular solution

T(m-1) u	0.20082
T(m+1) u	0.19988
T(m-1) v	0.31086
T(m+1) v	0.30956
d	0.00
x	0.00
CCT	6503.70
duv	0.00322
CCT	**6504.39**

0.2004	0.3103
Sign of Duv	1
Duv	**0.00322**

0.15 % step table

	u	v	Distance
6429.7	0.20082	0.31086	0.003279
6438.9	0.20077	0.31080	0.003263
6448.1	0.20072	0.31073	0.003250
6457.4	0.20067	0.31066	0.003238
6466.6	0.20062	0.31060	0.003229
6475.8	0.20057	0.31053	0.003222
6485.0	0.20053	0.31047	0.003216
6494.3	0.20048	0.31040	0.003213
6503.5	0.20043	0.31034	0.003213
6512.7	0.20038	0.31027	0.003214
6522.0	0.20034	0.31021	0.003217
6531.2	0.20029	0.31014	0.003222
6540.4	0.20024	0.31008	0.003229
6549.6	0.20020	0.31001	0.003239
6558.9	0.20015	0.30995	0.003250
6568.1	0.20010	0.30988	0.003263
6577.3	0.20006	0.30982	0.003278
6586.6	0.20001	0.30976	0.003295
6595.8	0.19997	0.30969	0.003313
6605.0	0.19992	0.30963	0.003334
6614.2	0.19988	0.30956	0.003356

minimum	0.0032125	
match	9	
T(m-1)	6494.3	0.00321
T(m)	6503.5	0.00321
T(m+1)	6512.7	0.00321

Parabolic solution

a	1.886E-05
b	-3.77E-05
c	1.886E-05
A	1.223E-08
B	-0.000159
C	0.5202378
CCT	6503.05
duv	0.00321

Triangular solution

T(m-1) u	0.20048
T(m+1) u	0.20038
T(m-1) v	0.31040
T(m+1) v	0.31027
d	0.00
x	0.00
CCT	6503.03
duv	0.00321
CCT	**6503.05**

0.2004	0.3103
Sign of Duv	1
Duv	**0.00321**

0.015 % step table

	u	v	Distance
6494.3	0.20048	0.31040	0.003213
6495.2	0.20047	0.31040	0.003213
6496.1	0.20047	0.31039	0.003213
6497.0	0.20046	0.31038	0.003213
6498.0	0.20046	0.31038	0.003213
6498.9	0.20045	0.31037	0.003213
6499.8	0.20045	0.31036	0.003213
6500.7	0.20045	0.31036	0.003213
6501.7	0.20044	0.31035	0.003213
6502.6	0.20044	0.31034	0.003213
6503.5	0.20043	0.31034	0.003213
6504.4	0.20043	0.31033	0.003213
6505.3	0.20042	0.31032	0.003213
6506.3	0.20042	0.31032	0.003213
6507.2	0.20041	0.31031	0.003213
6508.1	0.20041	0.31030	0.003213
6509.0	0.20040	0.31030	0.003213
6510.0	0.20040	0.31029	0.003213
6510.9	0.20039	0.31028	0.003213
6511.8	0.20039	0.31028	0.003213
6512.7	0.20038	0.31027	0.003214

minimum	0.0032125	
match	10	
T(m-1)	6501.7	0.00321
T(m)	6502.6	0.00321
T(m+1)	6503.5	0.00321

Parabolic solution

a	0.001886
b	-0.003772
c	0.001886
A	1.22E-08
B	-0.000159
C	0.520744
CCT	6503.03
duv	0.00321

Triangular solution

T(m-1) u	0.20044
T(m+1) u	0.20043
T(m-1) v	0.31035
T(m+1) v	0.31034
d	0.00
x	0.00
CCT	6503.05
duv	0.00321
Final CCT	**6503.03**

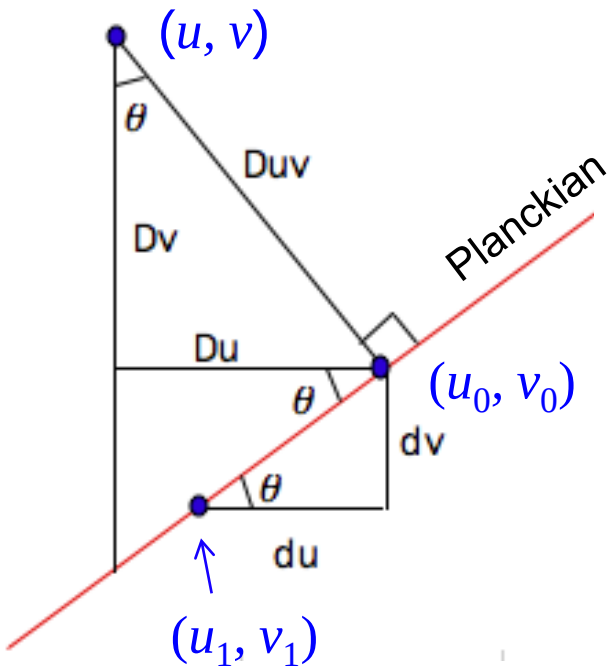
0.2004	0.3103
Sign of Duv	1
Final Duv	**0.00321**

Used as the reference for accuracy verification.

Conversion from (CCT, Duv) back to (x, y)

Input: CCT T (K)
Duv D_{uv}

1) Calculate (u_0, v_0) of the Planckian radiator at T (K).
2) Calculate (u_1, u_1) of the Planckian radiator at T+DT (K). DT=0.01 (K)
3) Calculate



$$du = u_1 - u_0$$

$$dv = v_1 - v_0$$

$$u = u_0 + D_{uv} \cdot \sin\theta$$

$$= u_0 + D_{uv} \cdot dv / \sqrt{du^2 + dv^2}$$

$$v = v_0 + D_{uv} \cdot \cos\theta$$

$$= u_0 + D_{uv} \cdot du / \sqrt{du^2 + dv^2}$$

$$u' = u$$

$$v' = 1.5\,v$$

$$x = 9u'/(6u' - 16v' + 12)$$

$$y = 2v'/(3u' - 8v' + 6)$$

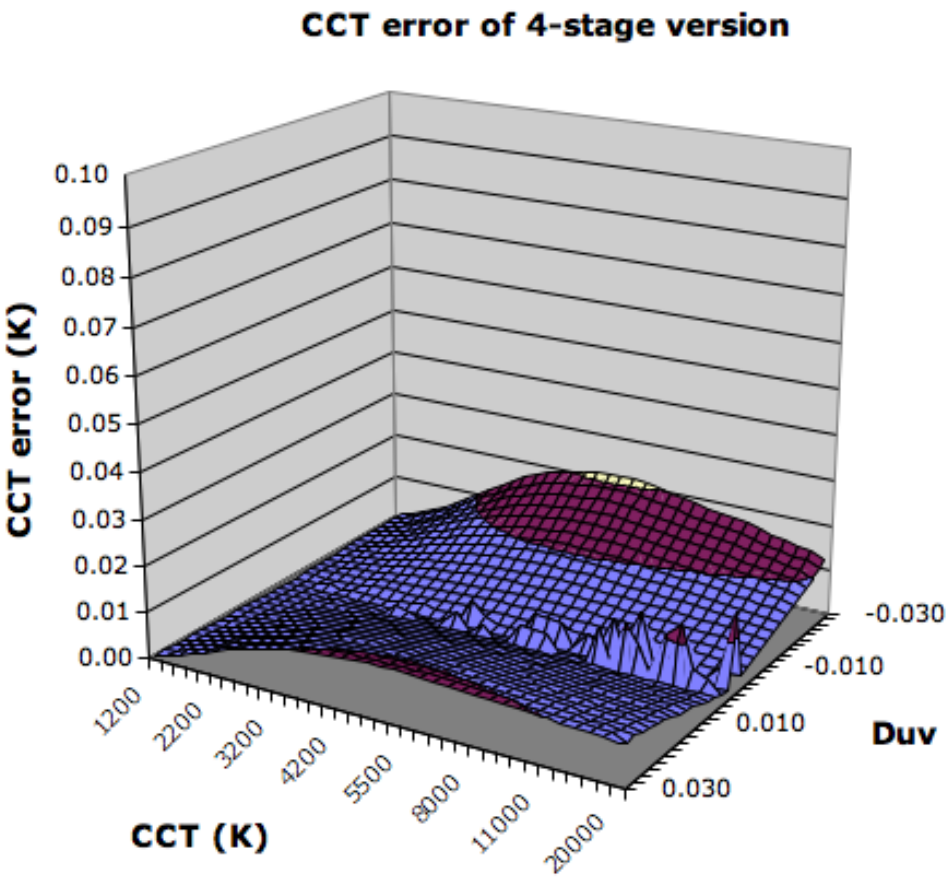
(Included in Revision draft of C78.377)

Accuracy of Most Accurate Version (4 stage)

Accuracy verification for 4-stage version

CCT	2900				
Duv	0.0200				
			OUTPUT	Error	
x	0.478420	CCT	2900.006	0.006	K
y	0.473737	Duv	0.020000	0.00000	
u	0.247629				
v	0.367808				

CCT 4-stage version



Simple calculation from (x,y) or (u’,v’) to Duv

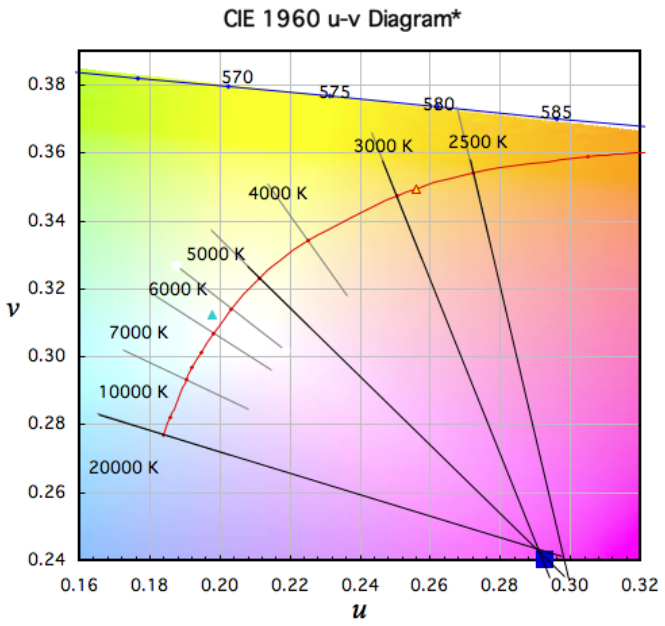
Duv is normally calculated in the process of calculating CCT.
Below is a simple approximation formula, without calculation of CCT.

1) Convert (x, y) or (u', v') to (u, v)

$$u = 4x/(-2x + 12y + 3)$$
$$v = 6y/(-2x + 12y + 3)$$

or

$$u = u'$$
$$v = 2v'/3$$



2) Duv is obtained by

$$L_{FP} = \sqrt{(u - 0.292)^2 + (v - 0.24)^2}$$

$$a = \arccos\left(\frac{u - 0.292}{L_{FP}}\right)$$

$$L_{BB} = k_6 a^6 + k_5 a^5 + k_4 a^4 + k_3 a^3 + k_2 a^2 + k_1 a + k_0$$

$$D_{uv} = L_{FP} - L_{BB}$$

k6	-0.00616793
k5	0.0893944
k4	-0.5179722
k3	1.5317403
k2	-2.4243787
k1	1.925865
k0	-0.471106

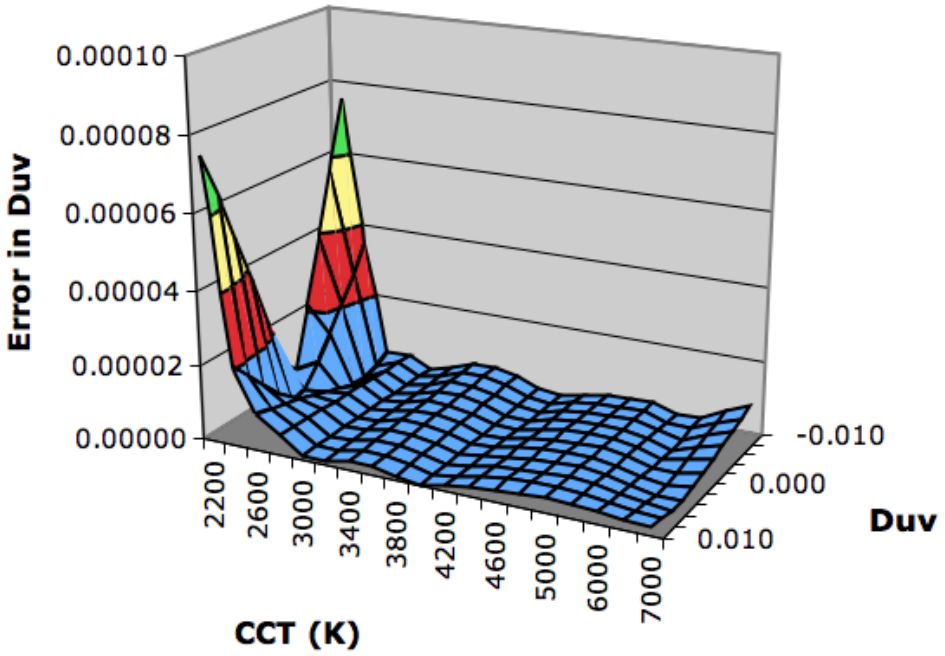
(Included in Revision draft of C78.377)

Simple calculation from (x,y) or (u’,v’) to Duv

Accuracy of this method

within 0.00001 in the range from 2600 K to 20000 K and Duv 0.000 ± 0.010

within 0.0001 in the range from 2160 K to 20000 K and Duv 0.000 ± 0.010



(Included in Revision draft of C78.377)

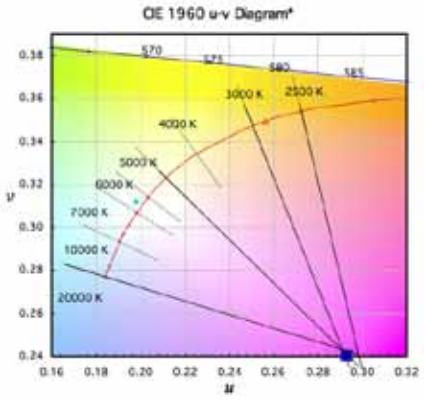
Simple calculation from (x,y) or (u’,v’) to (CCT, Duv)

$$L_{FP} = \sqrt{(u-0.292)^2 + (v-0.24)^2}$$

$$a_1 = \arctan((v-0.24)/(u-0.292)),\ \ if\ \ a_1 \geq 0,\ \ a = a_1;\ \ \ if\ \ a_1 < 0,\ \ a = a_1 + \pi$$

$$L_{BB} = k_{06}\,a^6 + k_{05}\,a^5 + k_{04}\,a^4 + k_{03}\,a^3 + k_{02}\,a^2 + k_{01}\,a + k_{00}$$

$$D_{uv} = L_{FP} - L_{BB}$$



For $a < 2.54;\ \ T_1 = 1/(k_{16}\cdot a^6 + k_{15}\cdot a^5 + k_{14}\cdot a^4 + k_{13}\cdot a^3 + k_{12}\cdot a^2 + k_{11}\cdot a + k_{10})$

For $a \geq 2.54;\ \ T_1 = 1/(k_{26}\cdot a^6 + k_{25}\cdot a^5 + k_{24}\cdot a^4 + k_{23}\cdot a^3 + k_{22}\cdot a^2 + k_{21}\cdot a + k_{20})$

For $a < 2.54;\ \ \Delta T_{c1} = (k_{36}\cdot a^6 + k_{35}\cdot a^5 + k_{34}\cdot a^4 + k_{33}\cdot a^3 + k_{32}\cdot a^2 + k_{31}\cdot a + k_{30}) * (L_{BB} + 0.01)/L_p * D_{uv}/0.01$

For $a \geq 2.54;\ \ \Delta T_{c1} = 1/(k_{46}\cdot a^6 + k_{45}\cdot a^5 + k_{44}\cdot a^4 + k_{43}\cdot a^3 + k_{42}\cdot a^2 + k_{41}\cdot a + k_{40}) * (L_{BB} + 0.01)/L_p * D_{uv}/0.01$

$$T_2 = T_1 - \Delta T_{c1},\ \ \ c = \log(T_2)$$

For $Duv \geq 0;\ \ \Delta T_{c2} = (k_{56}\cdot c^6 + k_{55}\cdot c^5 + k_{54}\cdot c^4 + k_{53}\cdot c^3 + k_{52}\cdot c^2 + k_{51}\cdot c + k_{50})$

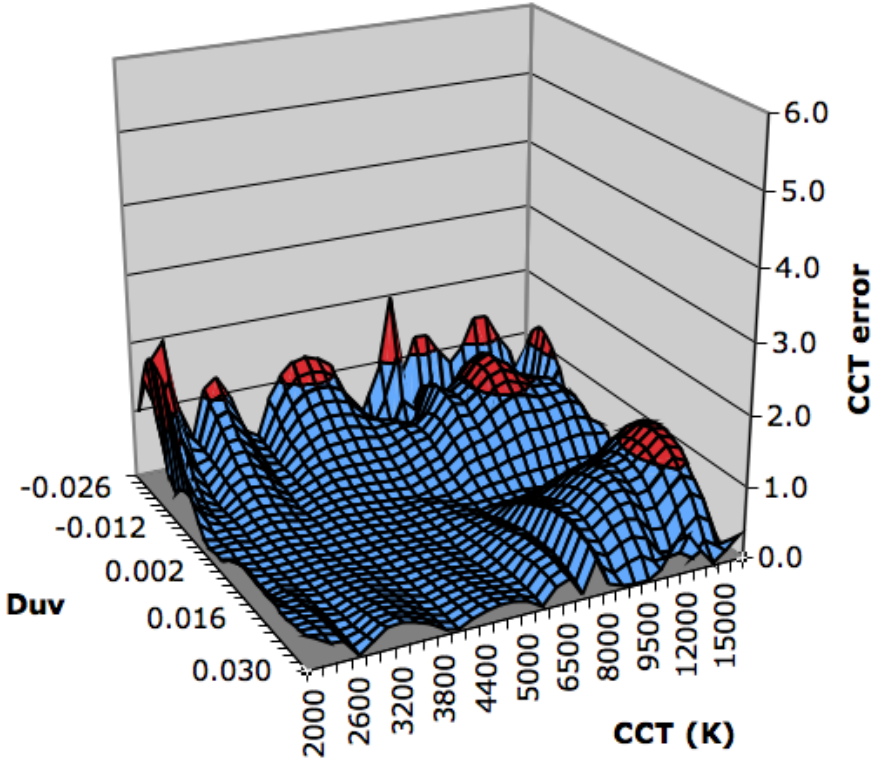
For $Duv < 0;\ \ \Delta T_{c2} = (k_{66}\cdot c^6 + k_{65}\cdot c^5 + k_{64}\cdot c^4 + k_{63}\cdot c^3 + k_{62}\cdot c^2 + k_{61}\cdot c + k_{60})\cdot \left|D_{uv}/0.03\right|^2$

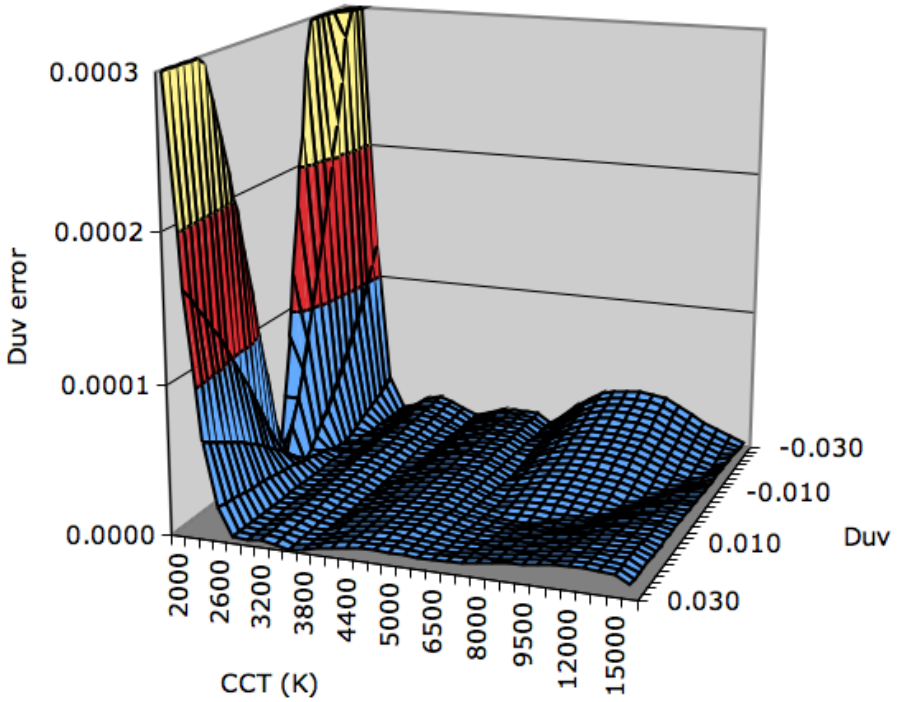
$$T_{FINAL} = T_2 - \Delta T_{c2}$$

i	ki6	ki5	ki4	ki3	ki2	ki1	ki0
0	-3.7146000E-03	5.6061400E-02	-3.307009E-01	9.750013E-01	-1.5008606E+00	1.115559E+00	-1.77348E-01
1	-3.2325500E-05	3.5700160E-04	-1.589747E-03	3.6196568E-03	-4.3534788E-03	2.1595434E-03	5.308409E-04
2	-2.6653835E-03	4.17781315E-02	-2.73172022E-01	9.53570888E-01	-1.873907584E+00	1.964980251E+00	-8.58308927E-01
3	-2.3524950E+01	2.7183365E+02	-1.1785121E+03	2.51170136E+03	-2.7966888E+03	1.49284136E+03	-2.3275027E+02
4	-1.731364909E+06	2.7482732935E+07	-1.81749963507E+08	6.40976356945E+08	-1.27141290956E+09	1.34488160614E+09	-5.926850606E+08
5	-9.4353083E+02	2.10468274E+04	-1.9500061E+05	9.60532935E+05	-2.65299138E+06	3.89561742E+06	-2.3758158E+06
6	5.0857956E+02	-1.321007E+04	1.4101538E+05	-7.93406005E+05	2.48526954E+06	-4.11436958E+06	2.8151771E+06

Simple calculation from (x,y) or (u’,v’) to (CCT, Duv)

Accuracy of this method





Conclusions

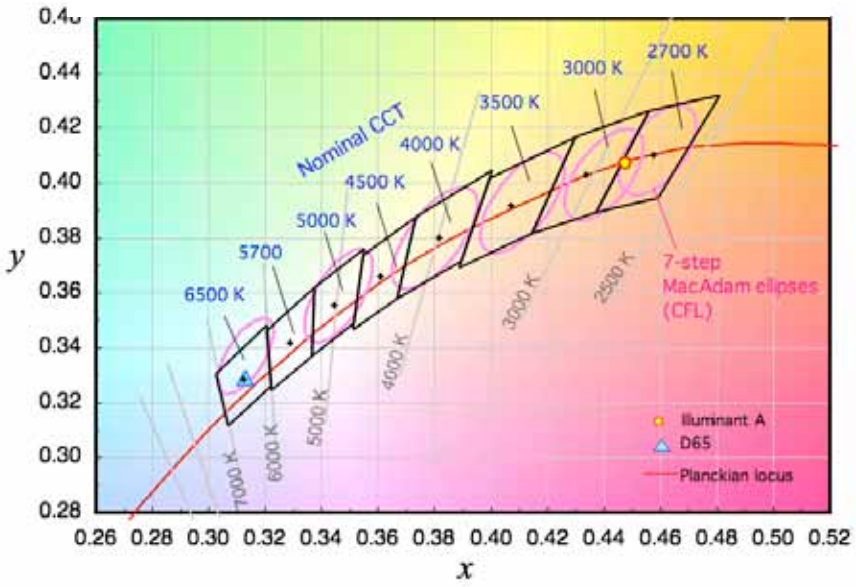
- Practical calculation and conversion formulae for CCT and Duv have been developed.
- Accuracies of some of the formulae will be further improved.
- The use of CCT and Duv (rather than x, y or u', v' chromaticity coordinates) is recommended to specify the chromaticity of lighting sources.

THANK YOU for your attention.

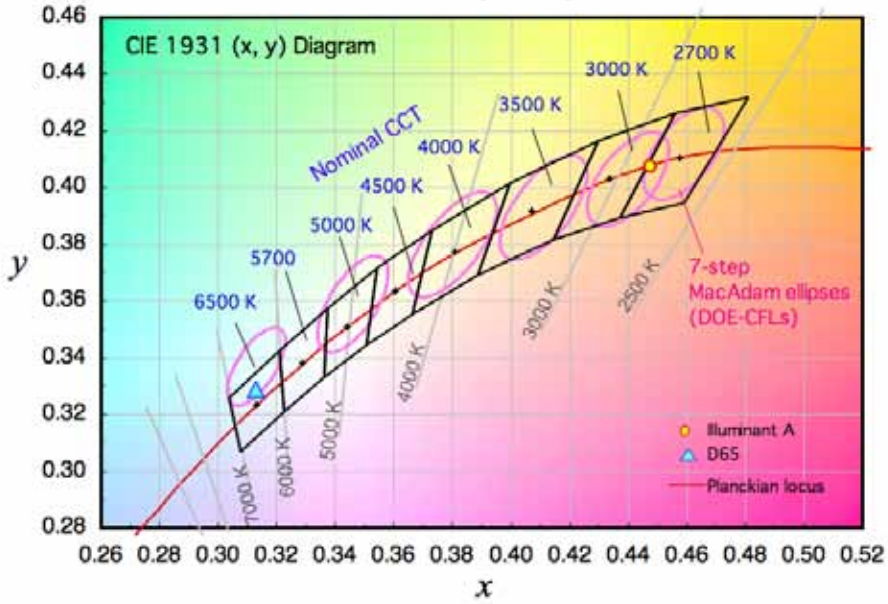
Contact: ohno@nist.gov

Proposed revision of ANSI C78.377

Current version

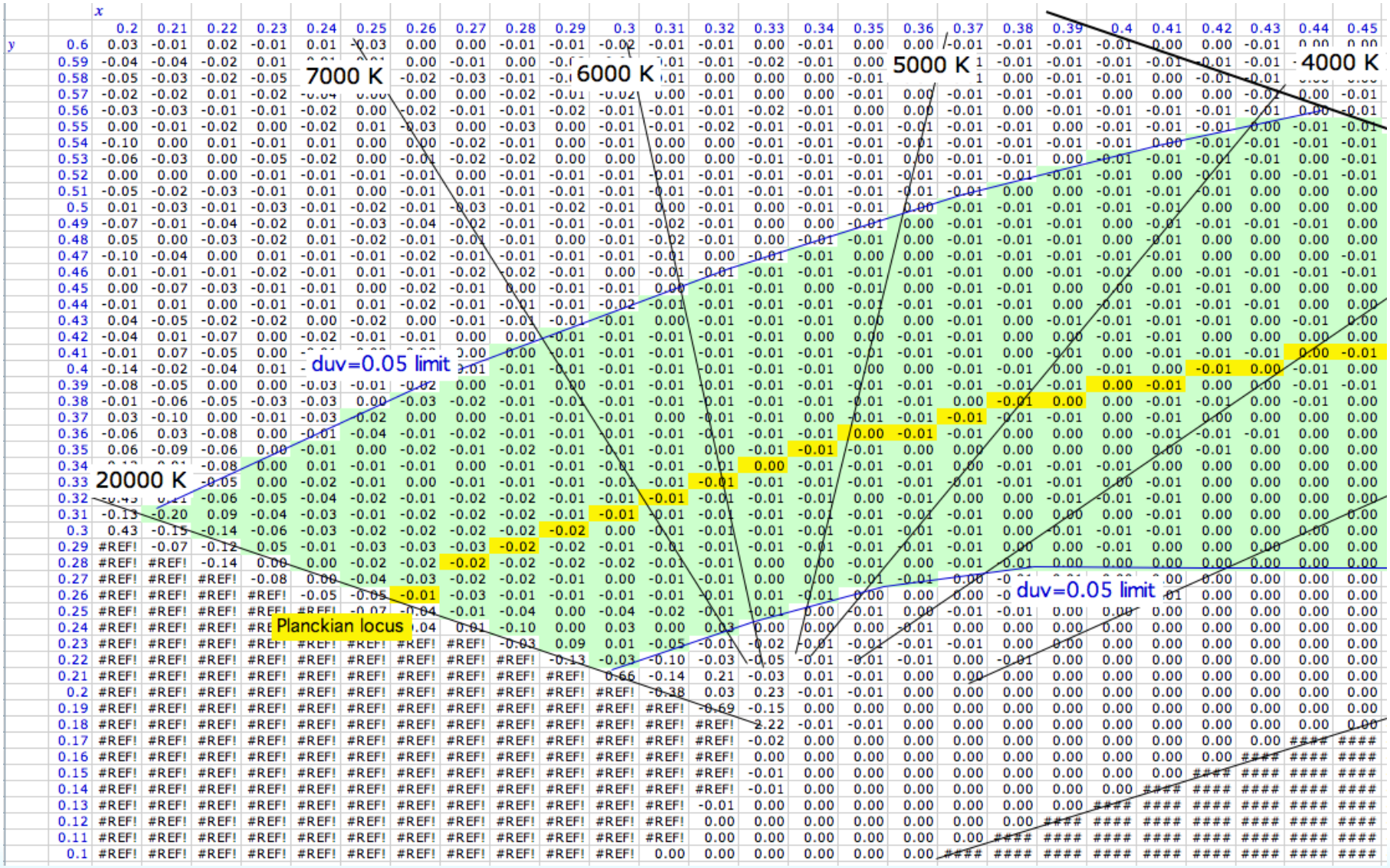


Revision proposal 2



- All center points to be moved onto the Planckian locus.
- This proposal is pending due to a need for vision experiments.
- Anecdotes say people prefer below the Planckian locus.
- NIST is funded by DOE to conduct vision experiments using STLF.

CCT (K) difference between 3rd stage and 5th stage



Summary

- Duv is important for color quality of light sources.
- Duv is often neglected in specifications.
- Parabolic and triangle combined solution works well for CCT calculation.
- 1 % step table provides enough accuracy
  (<1 K for 1000 to 10000 K, <2 K up to 20000 K, Duv±0.03)
- Most Accurate Version (cascade expansion),
- Conversion from (CCT, Duv) back to (*x,y*),
- Simple calculation from (*x,y*) or (*u',v'*) to Duv,
- Simple calculation from (*x,y*) or (*u',v'*) to (CCT, Duv) have been developed.